Bridging the gap between renormalization group and numerical simulations

Luka Sinković[1], mentored by Ivan Balog[2]
[1], [2] Institute of physics, Zagreb

Abstract

To study the criticality in disordered systems we have two main approaches. First is the renormalization group (RG) approach, which is the natural analytical framework for studying the criticality of any kind. It basically gives us an exact solution to the studied model. The conclusions we can thus obtain speak about an ideal infinite system directly at the criticality. To obtain any quantitative information about the model from RG, we need to resort to approximations within the RG approach which can be tricky as we have the posibility of destroying the physics we want to observe in the process. Second approach is numerical simulation, what we do is take a model with its microscopic Hamiltonian, place it in a finite lattice and simulate it. Here the theory of finite size scaling gives us a way of studying criticality from numerical data obtained in finite systems when extrapolated into size infinity. Because these two approaches are intrinsically different, there exists a huge chasm between the language and the questions asked in both approaches. The first objective of this work is to determine numerically exactly the quantities that are calculated in renormalization group such as the effective potential or the field renormalization function. This correspondence will be established on the relatively simple Ising model.

After the correspondence with renormalization group is established, we want to give new insight into the famous dimensional reduction (DR) puzzle. The DR is a famous puzzle in the statistical physics of phase transitions originating from the work of Parisi and Sourlas[1] on random field Ising model which claims that the critical behavior of the d dimensional RFIM should be completely equivalent to the critical behavior of the $d-2$ dimensional pure Ising model. Even though this claim seems too elegant, it took years until Aizenman and Wehr[2] have shown that dimensional reduction has to be wrong for low enough dimensions due to the fact that it was found that lower critical dimension for RFIM is $d = 2$. We know that lower critical dimension of pure Ising model is $d = 1$, and that dimensional reduction in this case does not stand.

A scenario about breaking of the DR was proposed recently[3], which claims that the dominance of avalanches for the critical behavior is responsible for DR breaking. This is connected with a singularity that appears in the effective disorder correlator. We wish to test numerically this scenario and compare it with the analytical prediction[4] that was found on a 1d long-range RFIM[5][6]. If the test turns out to be positive, this is a strong argument that the scenario holds correct also in the original model.

Keywords— random field Ising model, long-range interactions, numerical calculations, ground state determination, renormalization group, dimensional reduction, cusp singularity

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