PASSIVE RC FILTERS

RC filters consists of a capacitor and a resistor combined in order to pass either the AC signal of lower frequencies with unattenuated amplitude (low-pass filter), while the higher frequencies are attenuated, or the AC signal of higher frequencies with unattenuated amplitude (high-pass filter), while the lower frequencies are attenuated .

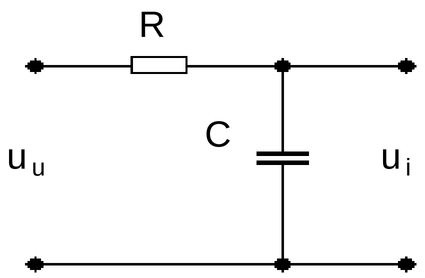


Figure 1a: *Low-pass (LP) filter*

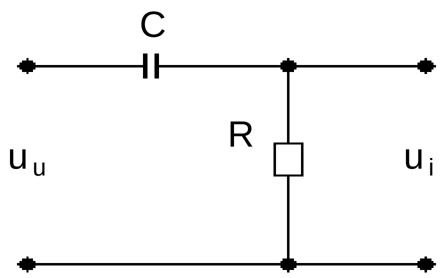


Figure 1b: *High-pass (HP) filter*

HIGH-PASS FILTER

This circuit can be considered as a frequency-dependent voltage divider (figure 1b). Ohmic resistance R and capacitive resistance 1/jωC determine the ratio of output and input voltage for a given frequency:

$$\frac{u_i}{u_u} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR} = \frac{\omega^2 R^2 C^2}{1 + \omega^2 R^2 C^2} + j\frac{\omega CR}{1 + \omega^2 R^2 C^2} \quad (1)$$

Ratio of voltage amplitudes of the signal on input and output are given by absolute value of this equation:

$$\frac{|u_i|}{|u_u|} = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 R^2 C^2}}} \quad (2)$$

Half-power frequency or frequency limit ν_g can be defined as a frequency at which the electric output power on a load resistor R_p falls to half the value of the input power that would be obtained on the same load resistor. This requirement yields:

$$\frac{1}{2}=\frac{P_i}{P_u}=\frac{\frac{|u_i|^2}{R_p}}{\frac{|u_u|^2}{R_p}} \quad \text{and the next equation follows:} \quad \frac{|u_i|}{|u_u|}=\frac{1}{\sqrt{2}} \qquad (3)$$

Using (2) and (3) we can find determine frequency limit (circular and linear frequency):

$$\omega_g=\frac{1}{RC}, \quad \nu_g=\frac{1}{2\pi RC} \qquad (4)$$

and:

$$\frac{|u_i|}{|u_u|}=\frac{1}{\sqrt{1+\frac{\nu_g{}^2}{\nu^2}}}$$

The above dependence is given in figure 4b.

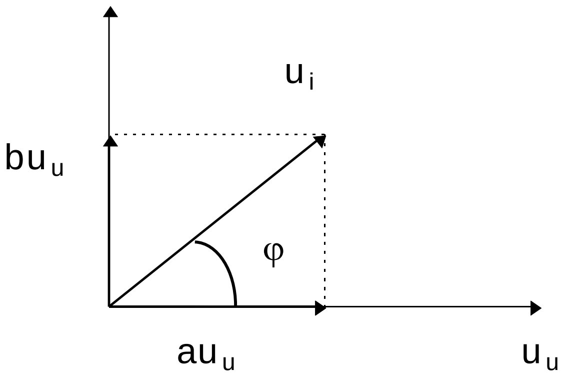
From equation (1), it follows that the ratio of the output and input voltage is in fact a complex ratio that depends on frequency. This means that a frequency shift φ must exist between output and input signal, and that this shift changes with frequency. Dependence of phase shift on frequency can be seen on phase characteristic $\varphi(\nu)$ of the filter. We can write output voltage (relation 1) as a complex number dependent on input voltage, with the ratio represented as a complex number:

$$u_i=(a+jb)\,u_u$$

from which

$$au_u=\frac{\omega^2R^2C^2}{1+\omega^2R^2C^2}u_u \quad \text{is a real part of the output signal, and}$$

$$bu_u=\frac{\omega\,R\,C}{1+\omega^2R^2C^2}u_u \quad \text{is imaginary part of the output signal.}$$



We can represent complex input and output voltage in a complex plane (figure 2). It can be seen that the phase angle φ between output and input voltage can be obtained as:

$$\varphi = \mathrm{arctg}\frac{b}{a}$$

Figure 2. *Phase shift of the output voltage compared to the input voltage*

If we put values a i b for HP filter in the above equation, we get:

$$\varphi = \mathrm{arc\ tg}\frac{1}{\omega RC} = \mathrm{arc\ tg}\frac{\omega_g}{\omega} \tag{5}$$

LOW-PASS FILTER

This filter (figure 1a) can also be considered as a voltage divider:

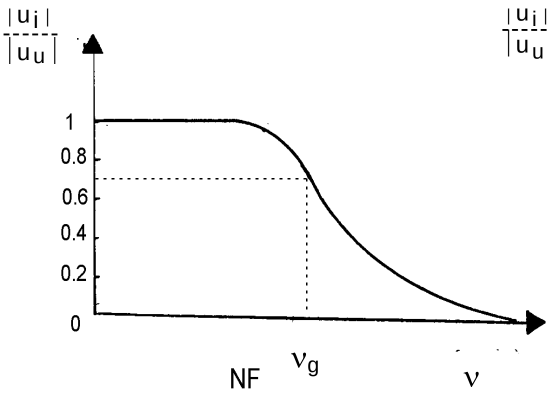
$$\frac{u_i}{u_u} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1+\omega^2 R^2 C^2} - j\frac{\omega RC}{1+\omega^2 R^2 C^2} \tag{6}$$

$$\frac{|u_i|}{|u_u|} = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \tag{7}$$

From (7) and (3), equation for limiting frequency can be obtained, which is the same as for high-pass filter:

$$\frac{|u_i|}{|u_u|} = \frac{1}{\sqrt{1+\frac{\nu^2}{\nu_g^2}}}$$

which can be seen in figure 4a.



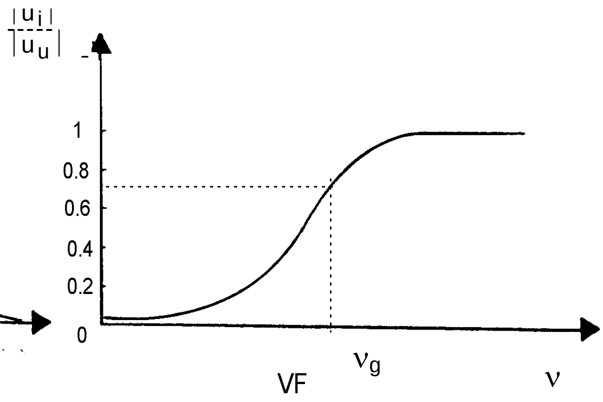


Fig. 4a: *Frequency characteristic of LP filter*   Fig. 4b: *Frequency characteristic of HP filter*

Phase shift of the output signal compared to input signal, $\varphi = \text{arc tg}\frac{b}{a}$, can be found for LP filter if we use real and imaginary part of the complex number in relation (6):

$$\varphi = \text{arc tg}\,(-\omega RC) \;=\; \text{arc tg}\,(-\frac{\omega}{\omega_g})$$

NARROW-BAND FILTER

We can obtain narrow-band filter from a combination of low-pass and high-pass filter consisting of the same values of capacitor C and resistor R. This filter will pass only very narrow interval of frequencies around the limiting frequency, and attenuate the signal of lower and higher frequencies. Both LP and HP filters consisting of the same capacitances and resistors have the same limiting frequency, which is then also the limiting frequency of the narrow-band filter. Figure 8 shows dependence of the ratio of the amplitudes of output and input voltages on the frequency.

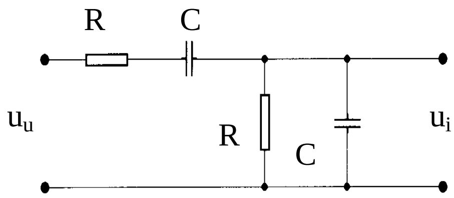


Figure 7. *Narrow-band filter*

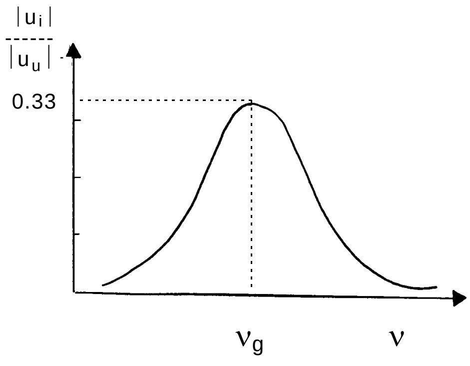


Figure 8. *Frequency characteristic of narrow-band filter*

As we did for LP and HP filter, narrow-band filter can be considered as a voltage divider:

$$\frac{u_i}{u_u} = \frac{\dfrac{1}{\dfrac{1}{R} + j\omega C}}{\dfrac{1}{\dfrac{1}{R} + j\omega C} + R + \dfrac{1}{j\omega C}} = \frac{j\omega RC}{1 + 3j\omega RC - \omega^2 R^2 C^2}$$

$$\frac{u_i}{u_u} = \frac{3\omega^2 R^2 C^2}{(1-\omega^2 R^2 C^2)^2 + 9\omega^2 R^2 C^2} + j\frac{\omega RC(1-\omega^2 R^2 C^2)}{(1-\omega^2 R^2 C^2)^2 + 9\omega^2 R^2 C^2} \qquad (8)$$

$$\frac{|u_i|}{|u_u|} = \frac{1}{\sqrt{(\dfrac{1}{\omega RC} - \omega RC)^2 + 9}} \qquad (9)$$

Frequency characteristic can be obtained from relation (9) and shown in figure 8, with a maximum at:

$$\frac{1}{\omega RC} - \omega RC = 0$$

or when $\omega RC = 1$, i.e. on frequency $\omega = \frac{1}{RC}$ which corresponds to the limiting frequency of LP and HP filters (4).

Phase shift of output signal compared to input signal, $\varphi = \mathrm{arctg}\frac{b}{a}$, can be obtained if we use imaginary and real part of the complex number of the ratio of the output and input voltages of the signal $\frac{u_i}{u_u} = f(\omega)$, which are found in relation (8).

$$\varphi = \mathrm{arctg}\frac{1-\omega^2 R^2 C^2}{3\omega RC} = \mathrm{arctg}(\frac{1}{3\omega RC} - \frac{\omega RC}{3}) = \mathrm{arctg}(\frac{\omega_g}{3\omega} - \frac{\omega}{3\omega_g})$$

for $\omega = \omega_g$ $\qquad \varphi = 0$

for $\omega << \omega_g$ $\qquad \varphi = \mathrm{arctg}\ (+\infty) = +\frac{\pi}{2}$

for $\omega >> \omega_g$ $\qquad \varphi = \mathrm{arctg}\ (-\infty) = -\frac{\pi}{2}$

BROAD-BAND FILTER

Broad-band filter can be obtained by combining low-pass and high-pass filter consisting of the different values of capacitor C and resistor R. This filter will pass broad interval of frequencies from the limiting frequency of the high-pass filter up to the limiting frequency of the low-pass filter, and attenuate the signal of lower and higher frequencies. LP and HP filters consisting of the different capacitances and resistors have also different limiting frequency, and limiting frequency of HP filter must be chosen to be lower than the limiting frequency of LP filter. Figure 10 shows dependence of the ratio of the amplitudes of output and input voltages (represented as the power) on the frequency.

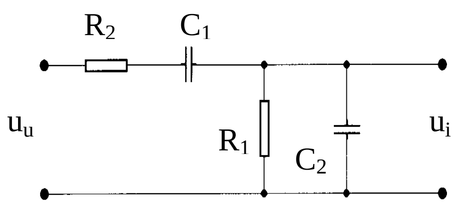


Figure 9. *Broad-band filter*

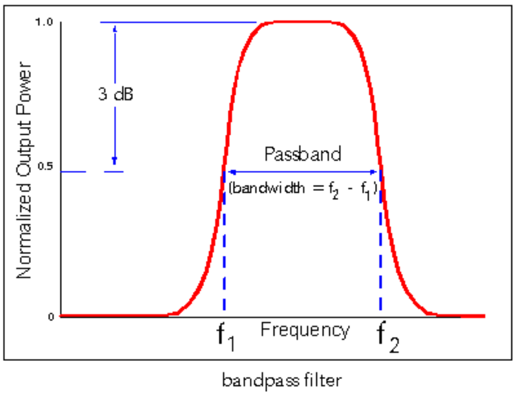


Figure 10. *Frequency characteristic of the broad-band filter*

On figure 10, frequency f_1 corresponds to the lower limiting frequency (high-pass component):

$$2\pi f_i = \omega_{VF} = \frac{1}{R_1 C_1}$$

while frequency f_2 corresponds to the higher limiting frequency (low-pass component):

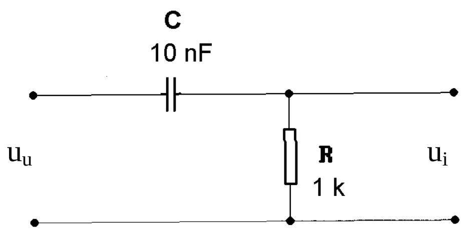
$$2\pi f_2 = \omega_{NF} = \frac{1}{R_2 C_2}$$

Filter bandwidth can be obtained as a frequency interval between the lower and higher limiting frequency:

$$\Delta\omega = \omega_{NF} - \omega_{VF}$$
$$\Delta f = f_2 - f_1$$

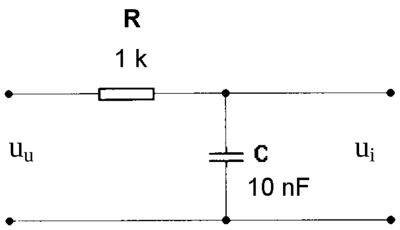
ASSIGNMENTS:

1. Connect function generator and tube voltmeter at the input of the high-pass filter, and tube voltmeter at the output (see the figure).



   a) Measure output voltage u_i for a given input sinusoidal voltage u_u = 5V.
   b) Measure and show on figure frequency characteristic $|u_i|/|u_u|$ = f (ν) as a ratio of output and input voltage amplitudes dependent on frequency.
   c) Determine limiting frequency and RC parameter from the frequency characteristic.
   d) Measure and show on figure phase characteristic $\varphi(\nu)$ of the high-pass filter using the two-channel oscilloscope, and compare it with the equation (5).

2. Assembly the low-pass filter with the elements as shown on the figure and do the same assignments as for the high-pass filter described in assignment 1.



3. Assembly the narrow-band filter by combining the low-pass and high-pass filters with the same values of R and C.
   a) Determine frequency characteristic and use it to obtain RC value of the given filter and the ratio of the amplitudes of the output and input signal in the maximum, and compare it with the computed expected value.
   b) Determine the phase characteristic by connecting input and output signal to the two-channel oscilloscope.

4. Assembly broad-band filter by combining the low-pass and high-pass filters according to the figure 9.
   a) Determine frequency characteristic and use it to find the values of the lower and higher limiting frequency, ratio of the amplitudes of the output and input

voltage in the maximum, and bandwidth (width of the frequency interval between lower and higher limiting frequency)

b) Determine the phase characteristic by connecting input and output signal to the two-channel oscilloscope.

INTEGRATION AND DERIVATION

DIFFERENTIATOR

It can be shown that high-pass filter (fig. 11) can be used as a differentiator of the input signal if, regardless of its shape, the following condition is fulfilled:

$$\nu \ll \nu_g$$

where ν_g is the limiting frequency of the HP filter.

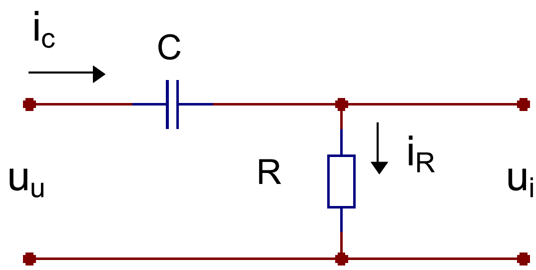


Fig. 11. *Voltage differentiator*

Output voltage u_i, which depends on input voltage u_u, is a solution of the general differential equation of the electrical circuit. Differential equation can be obtained from input and output circuit of the voltage differentiator (fig. 11):

from $\quad u_i = i_R R; \qquad i_R = i_C; \qquad u_C = u_u - u_i$

where u_C is voltage across the capacitor C, u_R is voltage across the resistor R, i_R and i_C are currents through resistor and capacitor, u_i and u_u are output and input voltages, follows:

$$i_c = \frac{dQ}{dt} = C\,\frac{du_C}{dt}$$

$$u_i = RC\frac{du_C}{dt} = RC\frac{d}{dt}(u_u - u_i)$$

where Q is the charge stored on the capacitor.

It can be seen that output voltage u_i will be proportional to the derivation of the input voltage du_u/dt if

$$u_i \ll u_u$$

because

$$u_i \approx RC\frac{du_u}{dt}$$

This means that the voltage across the capacitor C is much larger than voltage across the resistor R. This is fulfilled if $R \ll |X_C|$, i.e.

$$R \ll \frac{1}{2\pi\nu C}$$

or

$$\nu << \frac{1}{2\pi RC}$$

RC parameter determines limiting frequency of the circuit according to relation (4), so the condition for derivative behaviour of the circuit follows:

$$\nu \ll \nu_g$$

If a rectangular signal with a duration of t_d, which corresponds to the frequency $\nu = \frac{1}{2t_d}$, is applied as an input, output signal will be derivative of the input signal if its frequency is $\nu \ll \nu_g$. Shapes of input and output voltages in a derivative circuit are shown on figure 13.

INTEGRATOR

Again, we can show that low-pass filter can be used to integrate an alternate input signal if we solve differential equation of the low-pass filter circuit (fig. 12). Differential equation can be obtained by the following procedure:

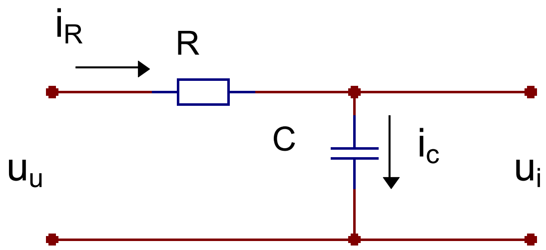


Figure 12. Voltage integrator

$$u_u = i_R R + u_i = i_C R + u_c = RC\frac{du_C}{dt} + u_C$$

We took into consideration that:

$$i_R = i_C; \qquad u_C = u_i$$

Where u_u and u_i are input and output voltages, i_R and i_C are current through the resistor and capacitor C, u_C is voltage across the capacitor. It follows that:

$$u_u = RC\frac{du_i}{dt} + u_i$$

or

$$RC\frac{du_i}{dt} = u_u - u_i$$

When

$$u_i \ll u_u$$

Output voltage will be proportionate to the integral of the input voltage:

$$RC\frac{u_i}{dt} \approx u_u$$
$$u_i \approx \frac{1}{RC}\int u_u dt$$

This condition will be fulfilled when the voltage across the capacitor C (output voltage) can be neglected compared to the voltage across the resistor R. This means that the capacitance must be much less that Ohmic resistance:

$$\frac{1}{2\pi\nu C} \ll R$$

and we finally obtain condition for integration:

$$\nu \gg \nu_g$$

Where the limiting frequency ν_g is given by relation (4).

Figure 13 shows output voltage obtained by integration of the rectangular input signal. Time variation of the voltage at the filter output in the derivative and integrative circuits is determined by capacitor as a key electronic element of the circuit. Charging and discharging of the capacitor determine the derivative and integrative properties. Typical time needed to charge/discharge the capacitor depends on the charge that can be stored on it, i.e. on its capacitance, and on the current that charge/discharge the capacitor, i.e. on the resistor through which the capacitor is charged/discharged.

Therefore, shapes of the output voltages shown in figure 13 are in fact parts of the time-voltage characteristic of the capacitor (figure 14 and 15) at high and low frequencies. Ideal time-voltage characteristic of the rectangular input signal after integration will be linear (triangle signal), but in reality it represents small part of the time-voltage characteristic of the charging capacitor that can be linearly approximated.

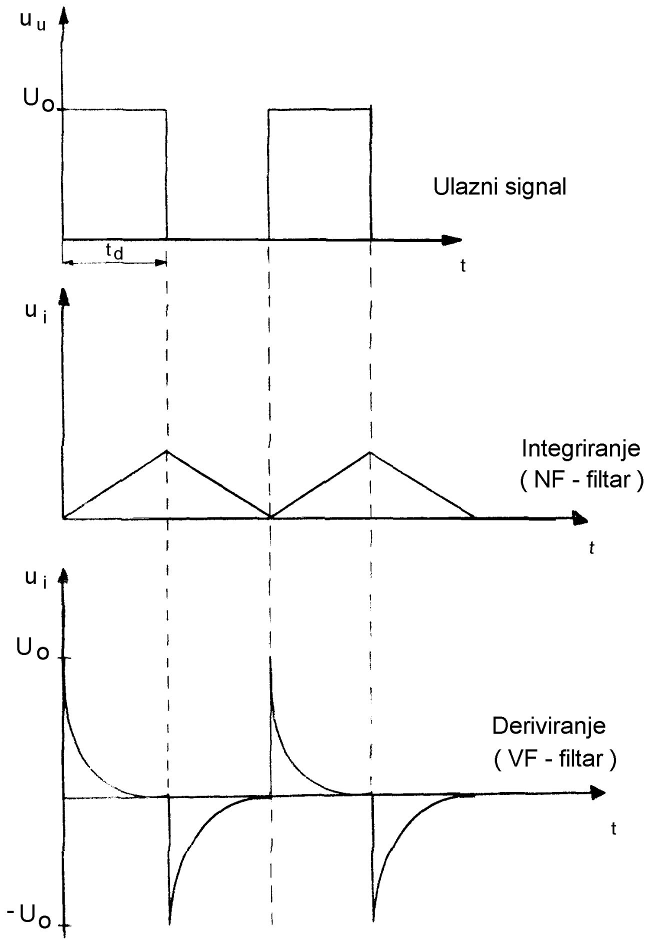


Figure 13. *Derivation and integration of the rectangular input signal*

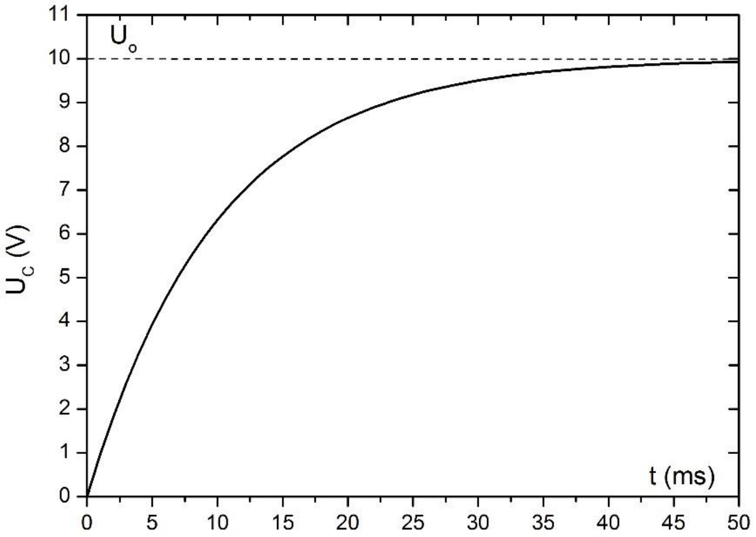


Figure 14. *Time-voltage characteristic of charging a capacitor (RC = 10 ms, U_0 = 10 V):*

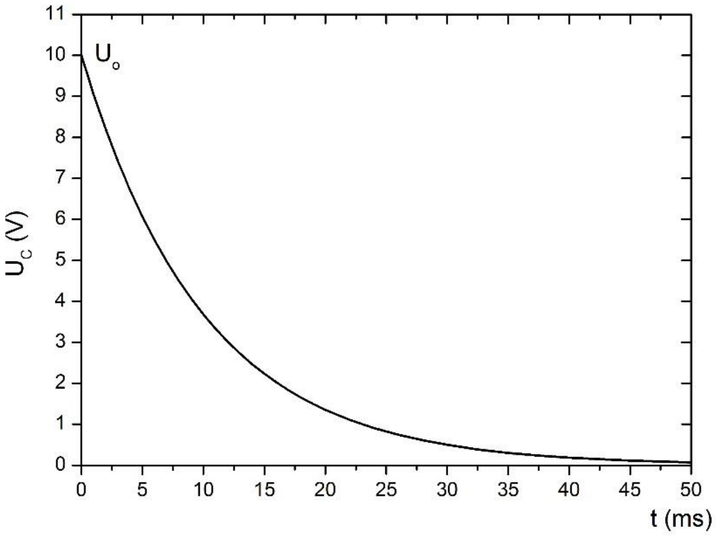


Figure 15. *Time-voltage characteristic of discharging a capacitor (RC = 10 ms, U_0 = 10 V)*

ASSIGNMENTS:

1. Connect input of the high-pass filter with the rectangular signal from the function generator. Observe dependence of output signal on the frequency of the input signal using the two-channel oscilloscope. Draw the shapes of the output signal for the following frequencies:

   $\nu = 0.1\ \nu_g$, $\nu = \nu_g$, $\nu = 10\ \nu_g$

   Determine the frequency for which this circuit differentiates the input signal.

2. Repeat assignments 1 but for low-pass filter. Determine the frequency for which LP filter circuit integrates the input signal.