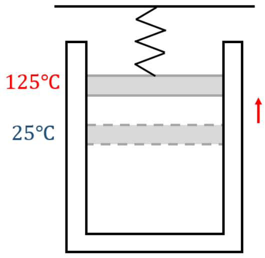
Fizika IV: Toplina i osnove statističke fizike

1. kolokvij - GRUPA A

25.5.2018.

1. Cilindar s pomičnim klipom napunjen je s 22,4 l plina pri temperaturi 25 °C i tlaku 1 atm. U takvom stanju klip spojimo s elastičnom zavojnicom koeficijenta 2000 Nm^{-1} prema slici.
   (a) Za koliko će se podignuti klip ako povisimo temperaturu na 125 °C? Površina klipa iznosi 0,01 m^2 i ima zanemarivu masu. (***2 boda***)
   (b) Koliki je omjer srednjih slobodnih putova molekula nakon pomaka klipa i prije pomaka klipa, ako se u cilindru nalazi vodik (H_2) koji se ponaša kao idealni plin. (***2 boda***)
   (c) Koliki je prirast unutrašnje energije? (***1,5 bod***)
   (d) Koji uvjet mora vrijediti za rad koji se obavi nad plinom u cilindru kako bi bilo moguće pomicanje klipa? (***0,5 boda***)



2. Cijev primarnog rashladnog kruga reaktora duljine 5 m izrađena je od čelika ($\lambda_1 = 40\,\mathrm{Wm^{-1}K^{-1}}$) debljine 2 cm i unutarnjeg promjera 1 m. S vanjske strane cijev je izolirana materijalom debljine 4 cm ($\lambda_2 = 0{,}02\,\mathrm{Wm^{-1}K^{-1}}$). Temperatura rashladnog sredstva iznosi 350 °C, a temperatura okoline 26,85 °C. Koliki je toplinski tok ako se pretpostavi da se unutarnja i vanjska strana cijevi održavaju na tim temperaturama? (***5 bodova***)

3. Mol zraka se zagrijava od 200 K do 800 K. Ovisnost molnog toplinskog kapaciteta pri konstantnom volumenu dana je izrazom $C_{\mathrm{m},V} = (15 + 0{,}05\,T)\,\mathrm{J\,mol^{-1}\,K^{-1}}$.
   (a) Kolika je promjena entropije ako je zagrijavanje izobarno? (***2 boda***)
   (b) Kolika je promjena entropije ako je zagrijavanje izohorno? (***1 bod***)

4. Za 1 mol plina koji zadovoljava jednadžbu stanja

$$\left(p + \frac{n^2}{V^2}\right) V = n\,R\,T$$

odredite koeficijente:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \qquad (\textit{\textbf{1 bod}})$$

$$\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V \qquad (\textit{\textbf{1 bod}})$$

i razliku $C_p - C_V$ (***1 bod***). Ako se stanje takvog plina mijenja od $\mathbf{A}(4\,\mathrm{K},\ 5\,\mathrm{m}^3)$ do $\mathbf{B}(4\,\mathrm{K},\ 2\,\mathrm{m}^3)$ izračunajte promjenu unutrašnje i (Helmholzove) slobodne energije. (***3 boda***).

Fizika IV: Toplina i osnove statističke fizike

1. kolokvij - GRUPA B

25.5.2018.

1. Za 1 mol plina koji zadovoljava jednadžbu stanja

$$\left(p + \frac{n^2}{V^2}\right) V = n\,R\,T$$

odredite koeficijente:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \qquad (\textit{\textbf{1 bod}})$$

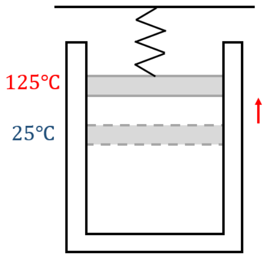
$$\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V \qquad (\textit{\textbf{1 bod}})$$

i razliku $C_p - C_V$ (***1 bod***). Ako se stanje takvog plina mijenja od $\mathbf{A}(4\,\mathrm{K},\ 3\,\mathrm{m}^3)$ do $\mathbf{B}(4\,\mathrm{K},\ 1\,\mathrm{m}^3)$ izračunajte promjenu unutrašnje i (Helmholzove) slobodne energije. (***3 boda***)

2. Mol zraka se zagrijava od 133 K do 331 K. Ovisnost molnog toplinskog kapaciteta pri konstantnom volumenu dana je izrazom $C_{\mathrm{m},V} = (17 + 0{,}03\,T)\,\mathrm{J\,mol^{-1}\,K^{-1}}$.
   (a) Kolika je promjena entropije ako je zagrijavanje izobarno? (***2 boda***)
   (b) Kolika je promjena entropije ako je zagrijavanje izohorno? (***1 bod***)

3. Cijev primarnog rashladnog kruga reaktora duljine 7 m izrađena je od čelika ($\lambda_1 = 40\,\mathrm{Wm^{-1}K^{-1}}$) debljine 8 cm i unutarnjeg promjera 80 cm. S vanjske strane cijev je izolirana materijalom debljine 10 cm ($\lambda_2 = 0{,}03\,\mathrm{Wm^{-1}K^{-1}}$). Temperatura rashladnog sredstva iznosi 400 °C, a temperatura okoline 16,85 °C. Koliki je toplinski tok ako se pretpostavi da se unutarnja i vanjska strana cijevi održavaju na tim temperaturama? (***5 bodova***)

4. Cilindar s pomičnim klipom napunjen je s 67,2 l plina pri temperaturi 25 °C i tlaku 1 atm. U takvom stanju klip spojimo s elastičnom zavojnicom koeficijenta 3000 Nm^{-1} prema slici.
   (a) Za koliko će se podignuti klip ako povisimo temperaturu na 125 °C? Površina klipa iznosi 0,02 m^2 i ima zanemarivu masu. (***2 boda***)
   (b) Koliki je omjer srednjih slobodnih putova molekula nakon pomaka klipa i prije pomaka klipa, ako se u cilindru nalazi dušik (N_2) koji se ponaša kao idealni plin. (***2 boda***)
   (c) Koliki je prirast unutrašnje energije? (***1,5 bod***)
   (d) Koji uvjet mora vrijediti za rad koji se obavi nad plinom u cilindru kako bi bilo moguće pomicanje klipa? (***0,5 boda***)



Fizika IV: Toplina i osnove statističke fizike

2. kolokvij - GRUPA A

17.6.2018.

1. Zadana je funkcija raspodjele $f(x) = C\,x\,e^{-b^4x^2}$ duž pozitivnog smjera osi x. Odredite srednju vrijednost koordinate i standardnu devijaciju $\sigma(x)$. (***5 bodova***)

2. Zadan je hamiltonijan jednodimenzionalnog sustava:

$$H = \frac{p^2}{2m} + Ax^2, \qquad 0 \leq x < \infty$$

   Kolika je prosječna energija sustava? Ako je zadana temperatura 300 K i količina tvari od jednog mola, koliko iznose unutrašnja energija U i toplinski kapacitet pri konstantnom volumenu C_V? (***5 bodova***)

3. Zadana je gustoća stanja fermionskog plina

$$g\,(E) = CVE^3 \qquad E \geq 0$$

   (a) odredite vjerojatnost da je energija fermiona na apsolutnoj nuli veća od prosječne energije, (***3 boda***)

   (b) odredite fluktuaciju energije čestice ΔE i relativnu flukutaciju energije čestice $\Delta E/\bar{E}$. (***3 boda***)

4. Kolika je vjerojatnost da je iznos x-komponente translacijske brzine između najvjerojatnije brzine i prosječne brzine? (***4 boda***)

Fizika IV: Toplina i osnove statističke fizike

2. kolokvij - GRUPA B

17.6.2018.

1. Kolika je vjerojatnost da je iznos x-komponente translacijske brzine između najvjerojatnije brzine i srednje kvadratne brzine? (***4 boda***)

2. Zadan je hamiltonijan jednodimenzionalnog sustava:

$$H = \frac{p^2}{2m} + Ax^2, \qquad 0 \leq x < \infty$$

   Kolika je prosječna energija sustava? Ako je zadana temperatura 150 K i količina tvari od dva mola, koliko iznose unutrašnja energija U i toplinski kapacitet pri konstantnom volumenu C_V? (***5 bodova***)

3. Zadana je funkcija raspodjele $f(z) = C\,z\,e^{-b^4 z^2}$ duž pozitivnog smjera osi z. Odredite srednju vrijednost koordinate i standardnu devijaciju $\sigma(z)$. (***5 bodova***)

4. Zadana je gustoća stanja fermionskog plina

$$g\,(E) = CVE^3 \qquad E \geq 0$$

   (a) odredite vjerojatnost da je energija fermiona na apsolutnoj nuli veća od prosječne energije, (***3 boda***)

   (b) odredite fluktuaciju energije čestice ΔE i relativnu flukutaciju energije čestice $\Delta E/\bar{E}$. (***3 boda***)

1. kolokvij

25.5.2018.

Ⓐ1. Ⓑ4.

Zadano: V_1, T_1, p_1, k

a) $\underline{T_2, S}$

$x = ?$

$$\frac{p_1 V_1}{T_1} = \frac{\overset{?}{p_2}\,\overset{?}{V_2}}{T_2}$$

- klip se pomiče prema gore: $V_2 > V_1 \rightarrow p_2 > p_1$

$$\left.\begin{aligned} V_2 &= V_1 + S \cdot x \\ p_2 &= p_1 + \frac{kx}{S} \end{aligned}\right\} \quad \frac{p_1 V_1}{T_1} = \frac{\left(p_1 + \frac{kx}{S}\right)(V_1 + Sx)}{T_2}$$

$$p_1 V_1 T_2 = T_1\left(p_1 V_1 \pm k V_1 \frac{x}{S} + p_1 S x + k x^2\right) / : T$$

$$\underbrace{k}_{=a} x^2 + \underbrace{\left(\frac{k V_1}{S} + p_1 S\right)}_{b} x + \underbrace{p_1 V_1 \left(1 - \frac{T_2}{T_1}\right)}_{=c} = 0$$

$$a x^2 + b x + c = 0$$

⋮

Ⓐ: $x_1 = 0{,}1322\,m$, $\cancel{x_2 < 0}$

Ⓑ: $x_1 = 0{,}1806\,m$, $\cancel{x_2 < 0}$

b) $\underline{H_2 \text{ (idealni plin)}, d}$

$$\frac{l_2}{l_1} = ? \longrightarrow \frac{l_2}{l_1} = \frac{\frac{1}{\sqrt{2}\, n_{02}\, \sigma}}{\frac{1}{\sqrt{2}\, n_{01}\, \sigma}} = \frac{n_{01}}{n_{02}} = \frac{\frac{N}{V_1}}{\frac{N}{V_2}} = \frac{V_2}{V_1} = \frac{V_1 + Sx}{V_1}$$

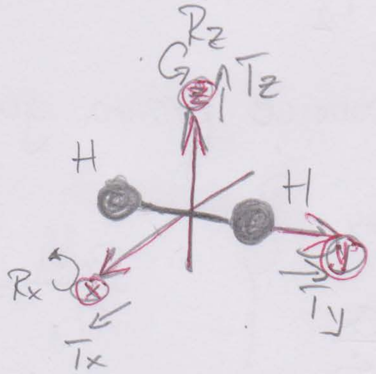
Ⓐ: $\frac{l_2}{l_1}=1{,}0591$

Ⓑ: $\frac{l_2}{l_1}=1{,}0537$

2. način

$$\left.\begin{aligned} n_0 &= \frac{N}{V} \\ pV &= NkT \;/ : VkT \\ \frac{N}{V} &= \frac{p}{kT} \end{aligned}\right\} \quad \frac{l_2}{l_1} = \frac{\frac{\cancel{k}T_2}{\cancel{\sqrt{2}}\cancel{\sigma} p_2}}{\frac{\cancel{k}T_1}{\cancel{\sqrt{2}}\cancel{\sigma} p_1}} = \frac{T_2 p_1}{T_1 p_2} = \frac{T_2 \cdot p_1}{T_1\left(p_1+\frac{kx}{S}\right)}$$

c) $\Delta U_{12}=?$



$f=5$ (dvoatomna molekula)

$$\underset{\sim}{U}=\frac{f}{2}NkT=\frac{f}{2}n\underbrace{N_A k}_{=R}T=\underbrace{\frac{f}{2}nRT}_{\downarrow\; n=?}$$

$p_1V_1=nRT_1 \;/ : RT_1$

$\boxed{n=\frac{p_1V_1}{RT_1}}$ ili $\boxed{n=\frac{p_2V_2}{RT_2}}$

$$\Delta U_{12}=\frac{5}{2}\frac{p_1V_1}{\cancel{R}T_1}\cancel{R}(T_2-T_1)$$

$\Delta U_{12}=\frac{5}{2}\frac{p_1V_1}{T_1}(T_2-T_1)$ ili $\Delta U_{12}=\frac{5}{2}\frac{p_2V_2}{T_2}(T_2-T_1)$

Ⓐ: $\Delta U_{12}=1903{,}14\,J$

Ⓑ: $\Delta U_{12}=5709{,}39\,J$

2. način:

$$\Delta U_{12} = \frac{f}{2}\underbrace{nR\Delta T_{12}}_{\Downarrow}$$

$$pV = nRT \,/ d$$

$$d(pV) = nRdT$$

$$\Delta(pV) = nR\Delta T$$

$$(p_2V_2 - p_1V_1) = nR(T_2 - T_1)$$

$$\Delta U_{12} = \frac{5}{2}(p_2V_2 - p_1V_1) =$$

3. način:

$$C_{m,V} = \frac{1}{n}\frac{dU}{dT} \Big/ \cdot ndT$$

$$dU = nC_{m,V}dT \Big/ \int$$

$$\int dU = \int nC_{m,V}dT$$

$$\Delta U = nC_{m,V}\Delta T$$

$$\Delta U_{12} = \frac{p_1V_1}{T_1}\cdot\frac{R}{\varkappa-1}(T_2 - T_1)$$

ili

$$\Delta U_{12} = \frac{p_2V_2}{T_2}\cdot\frac{R}{\varkappa-1}(T_2 - T_1)$$

$$C_{m,p} - C_{m,V} = R \Big/ : C_{m,V}$$

$$\varkappa - 1 = \frac{R}{C_{m,V}} \Big/ \cdot \left(\frac{C_{m,V}}{\varkappa-1}\right)$$

$$\boxed{C_{m,V} = \frac{R}{\varkappa-1}}$$

d) $\delta W = ?$

$W_{12} = ?$

$V_2 - V_1 > 0$

$\Delta V > 0$

$\underline{W_{12} > 0}$ tj. $\delta W > 0$

Ⓐ 2. i Ⓑ 3.

Zadano: $\underline{l,\ \lambda_1,\ x_1 = \Delta r_{12},\ d_1,\ \lambda_2,\ x_2 = \Delta r_{23},\ T_1,\ T_3}$

$\Phi = ?$ $\rightarrow$ uvjet: $\boxed{\Phi_{\check{c}} = \Phi_i = \Phi}$

$r_1 = \frac{d_1}{2}$

$r_2 = r_1 + x_1 = \frac{d_1}{2} + \Delta r_{12}$

$r_3 = r_2 + x_2 = \frac{d_1}{2} + \Delta r_{12} + \Delta r_{23}$

$$\left.\begin{aligned} \Phi_{\check{c}} &= \frac{2\pi \lambda_1 l (T_1 - T_2)}{\ln \frac{r_2}{r_1}} \\ \Phi_i &= \frac{2\pi \lambda_2 l (T_2 - T_3)}{\ln \frac{r_3}{r_2}} \end{aligned}\right\} \quad \Phi_{\check{c}} = \Phi_i = \Phi$$

Ⓐ: $T_2 = 623{,}065\ K$

$\Phi = 2739{,}08\ W$

Ⓑ: $T_2 = 672{,}873\ K$

$\Phi = 2672{,}88\ W$

2. način (bez računanja T_2)

$$\Phi_{\check{c}} = \frac{2\pi\lambda_1 (T_1 - T_2)}{\ln\frac{r_2}{r_1}} \rightarrow T_1 - \cancel{T_2} = \frac{\Phi_{\check{c}} \cdot \ln\frac{r_2}{r_1}}{2\pi\lambda_1}$$

$$\Phi_i = \frac{2\pi\lambda_2 (T_2 - T_3)}{\ln\frac{r_3}{r_2}} \rightarrow \cancel{T_2} - T_3 = \frac{\Phi_i \cdot \ln\frac{r_3}{r_2}}{2\pi\lambda_2}$$

$+$

$$\vdots$$

$$\Phi = \frac{2\pi l (T_1 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{\lambda_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{\lambda_2}}$$

3. način - preko toplinskih otpora 2 stjenke u "serijskom spoju"

$$R = R_{\check{c}} + R_i$$

$$R_{\check{c}} = -\frac{\Delta T}{\Phi_{\check{c}}} = -\frac{T_2 - T_1}{\Phi_{\check{c}}} = \frac{T_1 - T_2}{\Phi_{\check{c}}} = \frac{\cancel{(T_1 - T_2)}}{\frac{2\pi\lambda_1 l \cancel{(T_1 - T_2)}}{\ln\frac{r_2}{r_1}}}$$

$$\left.\begin{aligned} R_{\check{c}} &= \frac{\ln\frac{r_2}{r_1}}{2\pi\lambda_1 l} \\ R_i &= \frac{\ln\frac{r_3}{r_2}}{2\pi\lambda_2 l} \\ R &= \frac{T_1 - T_3}{\Phi} \end{aligned}\right\} \quad \begin{aligned} R &= R_{\check{c}} + R_i \\ \frac{T_1 - T_3}{\Phi} &= \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1 l} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2 l} \\ &\vdots \\ \Phi &= \frac{2\pi l (T_1 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{\lambda_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{\lambda_2}} \end{aligned}$$

Ⓐ 3. i Ⓑ 2.

Zadano: $n, T_1, T_2, \underset{\sim\sim\sim\sim}{C_{m,V} = f(T)}$

a) $p_1 = p_2 = p = konst.$

$$dS = \frac{\delta Q}{T} \Big/ \int$$

$$\int dS = \int \frac{\delta Q}{T}$$

$$\Delta S_{12} = \int_{T_1}^{T_2} \frac{\delta Q}{T}$$

$$\delta Q = dU + \delta W$$

$$\underline{dU = n \cdot C_{m,V} \cdot dT}$$

$$= pdV$$

$$p \cdot V = nRT / d$$

$$\underline{pdV = nRdT}$$

$$\boxed{\delta Q = n(C_{m,V} dT + RdT)}$$

$$\Delta S_{12} = \int_{T_1}^{T_2} \frac{n(C_{m,V} dT + RdT)}{T}$$

$$\Delta S_{12} = n \cdot \left\{ \int_{T_1}^{T_2} \frac{a + b \cdot T}{T} dT + \int_{T_1}^{T_2} \frac{R}{T} dT \right\}$$

$$\Delta S_{12} = n \left\{ \int_{T_1}^{T_2} \frac{a}{T} dT + \int_{T_1}^{T_2} b dT + \int_{T_1}^{T_2} \frac{R}{T} dT \right\}$$

$$\Delta S_{12} = n \left\{ a \ln \frac{T_2}{T_1} + b(T_2 - T_1) + R \ln \frac{T_2}{T_1} \right\}$$

$$\boxed{\Delta S_{12} = n \{ (a + R) \ln \frac{T_2}{T_1} + b(T_2 - T_1) \}}$$

Ⓐ $\Delta S_{12} = 62{,}32\ \mathrm{J/K}$

Ⓑ: $\Delta S_{12} = 29{,}02\ \mathrm{J/K}$

b) $V_1 = V_2 = V = \text{konst}$

$$dS = \frac{\delta Q}{T} \quad / \int$$

⋮

$$\Delta S_{12} = \int_{T_1}^{T_2} \frac{\delta Q}{T}$$

$$\delta Q = dU + \delta W$$

$$\delta W = \underline{\underline{0}}$$

$$\underline{dU = n C_{m,v}\, dT}$$

$$\boxed{\delta Q = n C_{m,v}\, dT}$$

$$\Delta S_{12} = \int_{T_1}^{T_2} \frac{n C_{m,v}\, dT}{T}$$

⋮

$$\boxed{\Delta S_{12} = n \left\{ a \ln \frac{T_2}{T_1} + b(T_2 - T_1) \right\}}$$

Ⓐ: $\Delta S_{12} = 50{,}79\ \mathrm{J/K}$

Ⓑ: $\Delta S_{12} = 21{,}44\ \mathrm{J/K}$

Ⓐ 4. i Ⓑ 1.

Zadano: $n = 1\,\text{mol}$

$$\left(p + \frac{n^2}{V^2}\right)V = nRT$$ ← jedn. stanja (<u>NIJE</u> idealni plin!!)

$$\left(p + \frac{n^2}{V^2}\right)V = nRT \;/\; d$$

$$d\left(p + \frac{n^2}{V^2}\right)V + \left(p + \frac{n^2}{V^2}\right)dV = nR\,dT$$

$$\left(dp - \frac{2n^2}{V^3}dV\right)V + \left(p\,dV + \frac{n^2}{V^2}dV\right) = nR\,dT$$

$$V\,dp - \frac{2n^2}{\underline{V^2}}dV + p\,dV + \frac{n^2}{\underline{V^2}}dV - nR\,dT = 0$$

$$V\,dp - nR\,dT + dV\left(p - \frac{n^2}{V^2}\right) = 0$$

a) $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = ?$

<u>p = konst:</u>

$$dV\left(p - \frac{n^2}{V^2}\right) = nR\,dT$$

$$\frac{dV}{dT} = \frac{nR}{p - \frac{n^2}{V^2}} = \left(\frac{\partial V}{\partial T}\right)_p \longrightarrow \boxed{\alpha = \frac{1}{V}\,\frac{nR}{p - \frac{n^2}{V^2}}}$$

$$\alpha = \frac{1}{\cancel{V}}\,\frac{nR}{\frac{pV^2 - n^2}{V^2}} = \frac{nRV}{pV^2 - n^2}$$

b) $\beta = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V = ?$

V=konst:

$$V dp = nR dT$$

$$\frac{dp}{dT} = \frac{nR}{V} = \left(\frac{\partial p}{\partial T}\right)_V \longrightarrow \boxed{\beta = \frac{1}{p}\frac{nR}{V} = \frac{nR}{pV}}$$

c) $C_p - C_V = ?$

$$C_p - C_V = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p - C_V = T\left(\frac{nR}{V}\right)\left(\frac{nR}{p-\frac{n^2}{V^2}}\right)$$

$$C_p - C_V = \frac{1}{\cancel{V}} \frac{Tn^2R^2}{\frac{pV^2-n^2}{V^{\cancel{2}}}}$$

$$\boxed{C_p - C_V = \frac{VTn^2R^2}{pV^2-n^2}}$$

---

2. način: $\left(p+\frac{n^2}{V^2}\right)V = nRT \,/d$

$$d\left(p+\frac{n^2}{V^2}\right)V + \underbrace{\left(p+\frac{n^2}{V^2}\right)}_{=\frac{nRT}{V}}dV = nRdT$$

$$Vdp - \frac{2n^2}{V^2}dV + \frac{nRT}{V}dV - nRdT = 0$$

$$Vdp - nRdT + dV\left(\frac{nRT}{V} - \frac{2n^2}{V^2}\right) = 0$$

⋮

isto kao i na 1. nacin:

a) $$\frac{dV}{dT} = \frac{\cancel{n}R}{\frac{\cancel{n}RT}{V} - \frac{2n^{\cancel{2}}}{V^2}} = \frac{R}{\frac{RTV-2n}{V^2}} = \frac{RV^2}{RTV-2n} = \left(\frac{\partial V}{\partial T}\right)_p$$

$$\boxed{\alpha = \frac{RV}{RTV-2n} = \frac{\cancel{RV}}{\cancel{RV}\left(T - \frac{2n}{RV}\right)} = \frac{1}{T - \frac{2n}{RV}}}$$

b) $$\frac{dp}{dT} = \frac{nR}{V} = \left(\frac{\partial p}{\partial T}\right)_V \longrightarrow \boxed{\beta = \frac{nR}{pV}}$$

c) $$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p - C_v = T\left(\frac{nR}{V}\right)\left(\frac{RV^2}{RTV-2n}\right)$$

$$C_p - C_v = \frac{TnR^2V^2}{RTV^2 - 2nV}$$

$$C_p - C_v = \frac{nR^{\cancel{2}}V^{\cancel{2}}\cancel{T}}{\cancel{RT}\cancel{V}^{\cancel{2}}\left(1 - \frac{2n}{RTV}\right)} =$$

$$\boxed{C_p - C_v = \frac{nR}{1 - \frac{2n}{RTV}}}$$

$$A(T_1, V_1) \longrightarrow B(T_2, V_2)$$

$\underline{T_1 = T_2 = T}$, $\underline{V_1 > V_2}$ $\Rightarrow$ izotermna kompresija 1 mola plina koji <u>NIJE</u> idealan!

d) $\Delta U_{AB} = ?$

$$dU = \cancel{C_V \underbrace{dT}_{=0}} + \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right] dV$$ ← OPĆA TERMODINAMIČKA JEDNADŽBA

$$dU = \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right] dV$$

iz jedn. stanja:

$$\left(p + \frac{n^2}{V^2}\right) V = nRT \,/ : V$$

$$\boxed{p = \frac{nRT}{V} - \frac{n^2}{V^2}}$$

$$dU = \left[\cancel{T\left(\frac{nR}{V}\right)} - \cancel{\frac{nRT}{V}} + \frac{n^2}{V^2}\right] dV$$

$$dU = \left(\frac{n^2}{V^2}\right) dV \,/ \int$$

$$\int_{U_0}^{U} dU = \int_{V_1}^{V_2} \frac{n^2}{V^2} dV$$

$$U - U_0 = n^2 \left(-\frac{1}{V}\right)\Big|_{V_1}^{V_2}$$

$$U - U_0 = n^2 \left(\frac{1}{V_1} - \frac{1}{V_2}\right) \rightsquigarrow U(V, T) = U_0(T) + \frac{1}{V_1} - \frac{1}{V_2}$$

$$\Delta U_{AB} = \frac{1}{V_1} - \frac{1}{V_2}$$

Ⓐ: $\Delta U_{AB} = -\frac{3}{10}\,J = -0{,}3\,J$

Ⓑ: $\Delta U_{AB} = -\frac{2}{3}\,J = -0{,}67\,J$

2. način:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$ ← JEDNADŽBA ZA UNUTRAŠNJU ENERGIJ

$$\left(\frac{\partial U}{\partial V}\right)_T = \cancel{T\left(\frac{nR}{V}\right)} - \cancel{\frac{nRT}{V}} + \frac{n^2}{V^2}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{n^2}{V^2}$$ ← desna strana ovisi samo o V

$$\frac{dU}{dV} = \frac{n^2}{V^2}$$

$$dU = \frac{n^2}{V^2}\,dV \Big/ \int$$

⋮

dalje isto kao na 1. način

e) $\Delta F_{AB} = ?$

$$F = U - TS \,/\, d$$

$$dF = dU - TdS - \underbrace{\cancel{SdT}}_{=0}$$

$$dF = dU - \underbrace{TdS}$$

$$TdS = \underbrace{\cancel{C_v dT}}_{=0} + T\left(\frac{\partial p}{\partial T}\right)_V dV$$ 1. TdS JEDNADŽBA

$$dF = dU - T\left(\frac{\partial p}{\partial V}\right)_V dV$$

$$dF = dU - T\frac{nR}{V}dV$$

$$dF = \frac{n^2}{V^2}dV - \frac{TnR}{V}dV \Big/ \int$$

$$\vdots$$

$$\Delta F_{AB} = \left(-\frac{1}{V}\right)\Big|_{V_1}^{V_2} - RT\ln(V)\Big|_{V_1}^{V_2}$$

$$\Delta F_{AB} = \frac{1}{V_1} - \frac{1}{V_2} - RT\ln\left(\frac{V_2}{V_1}\right)$$

Ⓐ: $\Delta F_{AB} = 30{,}17\,J$

Ⓑ: $\Delta F_{AB} = 35{,}87\,J$

2. način:

$$F = U - TS \,/\, d$$

$$dF = dU - TdS - \underbrace{\cancel{SdT}}_{=0}$$

$$dF = dU - TdS$$

$$\Delta F_{AB} = \Delta U_{AB} - T\Delta S_{AB}$$

$$\Delta S_{AB} = nR\ln\left(\frac{V_2}{V_1}\right) + \underbrace{n\,C_{m,V}\ln\underbrace{\left(\frac{T_2}{T_1}\right)}_{=1 \text{ jer je } T_1=T_2=T}}_{=0}$$

$$\Delta S_{AB} = nR\ln\left(\frac{V_2}{V_1}\right) = R\ln\left(\frac{V_2}{V_1}\right)$$

$$\Downarrow$$

$$\Delta F_{AB} = \Delta U_{AB} - T\Delta S_{AB}$$

$$\Delta F_{AB} = \frac{1}{V_1} - \frac{1}{V_2} - RT\ln\left(\frac{V_2}{V_1}\right)$$

2. kolokvij

17. 6. 2018.

Ⓐ 1. Ⓑ 3.

Zadano: $f(x) = Cxe^{-b^4x^2}$, $x \geq 0$

a) $\bar{x} = ?$ b) $G(x) = ?$

Ⓑ grupa: varijabla z umjesto x-a

a) $\bar{x} = \int_0^\infty x f(x)\,dx$

$$\bar{x} = \int_0^\infty x\, Cxe^{-b^4x^2}\,dx$$

→ normalizacija vjerojatnosti:

$$\int_0^\infty f(x)\,dx = 1$$

$$\int_0^\infty Cxe^{-b^4x^2}\,dx = 1$$

SUPSTITUCIJA

$t^2 = b^4x^2 /\sqrt{\ } \longrightarrow x^2 = \frac{t^2}{b^4}$

$t = b^2x \longrightarrow x = \frac{t}{b^2} /d$

$dx = \frac{dt}{b^2}$

$$\int_0^\infty C\cdot\frac{t}{b^2}e^{-t^2}\frac{dt}{b^2} = 1$$

$$\frac{C}{b^4}\underbrace{\int_0^\infty te^{-t^2}dt}_{I_1 = \frac{1}{2a} = \frac{1}{2}} = 1 \longrightarrow \frac{C}{2b^4} = 1 \Rightarrow \underline{\underline{C = 2b^4}}$$

2. način (brži)

$$\int_0^\infty Cxe^{-b^4x^2}dx = 1$$

$$C\underbrace{\int_0^\infty xe^{-(b^4)x^2}dx}_{I_1 = \frac{1}{2a} = \frac{1}{2b^4}} = 1 \longrightarrow \frac{C}{2b^4} = 1 \Longrightarrow \underline{\underline{C = 2b^4}}$$

$$\bar{x} = \int_0^\infty x \cdot 2b^4 x\, e^{-b^4x^2}dx$$

$$\bar{x} = 2b^4 \underbrace{\int_0^\infty x^2 e^{-(b^4)x^2}dx}_{I_2 = \frac{2-1}{2a}\cdot I_0 = \frac{2-1}{2a}\cdot\left(\frac{1}{2}\sqrt{\frac{\pi}{a}}\right) = \frac{1}{2b^4}\cdot\frac{1}{2}\sqrt{\frac{\pi}{b^4}} = \frac{\sqrt{\pi}}{4b^6}}$$

$$\bar{x} = \cancel{2}\cancel{b^4}\cdot\frac{\sqrt{\pi}}{\underset{2}{4}\,b^{\cancel{6}\,2}}$$

$$\boxed{\bar{x} = \frac{\sqrt{\pi}}{2b^2}}$$

b) $G(x) = \sqrt{\overline{x^2} - \bar{x}^2}$

$$\hookrightarrow (\bar{x})^2 = \left(\frac{\sqrt{\pi}}{2b^2}\right)^2 = \frac{\pi}{4b^4}$$

$$\downarrow$$

$$\overline{x^2} = \int_0^\infty x^2 f(x)\,dx$$

$$\overline{x^2} = \int_0^\infty x^2\, 2b^4 x\, e^{-b^4x^2}dx$$

$$\overline{x^2} = 2b^4\underbrace{\int_0^\infty x^3 e^{-(b^4)x^2}dx}_{I_3 = \frac{3-1}{2a}\cdot I_1 = \frac{\cancel{2}}{2b^4}\cdot\frac{1}{\cancel{2}b^4} = \frac{1}{2b^8}}$$

$$\overline{x^2} = \cancel{2}\cancel{b^4} \cdot \frac{1}{\cancel{2}b^{\cancel{8}4}}$$

$$\overline{x^2} = \frac{1}{b^4}$$

$$\sigma(x) = \sqrt{\frac{1}{b^4} - \frac{\pi}{4b^4}}$$

$$\sigma(x) = \sqrt{\frac{4-\pi}{4b^4}}$$

$$\boxed{\sigma(x) = \frac{\sqrt{4-\pi}}{2b^2} = 0{,}463\, b^{-2}}$$

Ⓐ: $\bar{x} = \frac{\sqrt{\pi}}{2b^2}$, $\sigma(x) = \frac{\sqrt{4-\pi}}{2b^2}$

Ⓑ: $\bar{z} = \frac{\sqrt{\pi}}{2b^2}$, $\sigma(z) = \frac{\sqrt{4-\pi}}{2b^2}$

Ⓐ 2. Ⓑ 2.

Zadano: $H=\frac{p^2}{2m}+Ax^2 \quad , \quad 0\le x<\infty$ $\boxed{\text{1D sustav}}$

a) $\overline{E}=?$

$$E=E_k+E_p=H$$

$$E_k=\frac{p^2}{2m}$$

$$E_p=Ax^2$$

$$\overline{E}=\underset{?}{\overline{E_k}}+\underset{?}{\overline{E_p}}$$

$\overline{E_k}$:

$$\overline{E_k}=\frac{\int E_k e^{-\beta E_k}d\phi}{\int e^{-\beta E_k}d\phi}=\frac{\int_0^\infty \frac{p^2}{2m}e^{-\frac{\beta p^2}{2m}}dp}{\int_0^\infty e^{-\frac{\beta p^2}{2m}}dp}= \left| \begin{array}{l} t^2=\frac{\beta}{2m}p^2 /\sqrt{\ } \longrightarrow p^2=\frac{2m}{\beta}t^2 \\ t=\sqrt{\frac{\beta}{2m}}\,p \longrightarrow p=\sqrt{\frac{2m}{\beta}}\,t \,/d \\ dp=\sqrt{\frac{2m}{\beta}}\,dt \end{array} \right|$$

$$\overline{E_k}=\frac{\int_0^\infty \frac{1}{\cancel{2m}}\left(\frac{\cancel{2m}}{\beta}t^2\right)e^{-t^2}\cancel{\sqrt{\frac{2m}{\beta}}}\,dt}{\int_0^\infty e^{-t^2}\cancel{\sqrt{\frac{2m}{\beta}}}\,dt}=\frac{1}{\beta}\,\frac{\int_0^\infty t^2e^{-t^2}dt}{\int_0^\infty e^{-t^2}dt}\begin{array}{l}\}\, I_2=\frac{2-1}{2a}\cdot I_0=\frac{\sqrt{\pi}}{4} \\ \}\, I_0=\frac{1}{2}\sqrt{\frac{\pi}{a}}=\frac{\sqrt{\pi}}{2}\end{array}$$

$$\boxed{\overline{E_k}=\frac{1}{\beta}\,\frac{\frac{\cancel{\sqrt{\pi}}}{4}}{\frac{\cancel{\sqrt{\pi}}}{2}}=\frac{1}{2\beta}=\frac{kT}{2}}$$

$\overline{E_p}$:

$$\overline{E_p}=\frac{\int E_p e^{-\beta E_p}d\phi}{\int e^{-\beta E_p}d\phi}=\frac{\int_0^\infty Ax^2e^{-\beta Ax^2}dx}{\int_0^\infty e^{-\beta Ax^2}dx}= \left| \begin{array}{l} v^2=\beta Ax^2 /\sqrt{\ } \longrightarrow x^2=\frac{v^2}{\beta A} \\ v=\sqrt{\beta A}\,x \longrightarrow x=\frac{v}{\sqrt{\beta A}} \,/d \\ dx=\frac{dv}{\sqrt{\beta A}} \end{array} \right|$$

$$\overline{E}_p = \frac{\int_0^\infty \cancel{A}\left(\frac{v^2}{\beta \cancel{A}}\right) e^{-v^2} \cancel{\left(\frac{1}{\sqrt{\beta A}}\right)} dv}{\int_0^\infty e^{-v^2} \cancel{\left(\frac{1}{\sqrt{\beta A}}\right)} dv} = \frac{1}{\beta} \frac{\left.\int_0^\infty v^2 e^{-v^2} dv \right\} I_2 = \frac{\sqrt{\pi}}{4}}{\left.\int_0^\infty e^{-v^2} dv \right\} I_0 = \frac{\sqrt{\pi}}{2}} = \frac{kT}{2}$$

$$\boxed{\overline{E} = \frac{kT}{2} + \frac{kT}{2} = kT}$$

b) Zadano T, n

$U = ?$ $C_V = ?$

$$U = N \cdot \overline{E} = n \cdot N_A \cdot \overline{E} = n \cdot \underbrace{N_A \cdot k}_{R} \cdot T = nRT$$ //

$$C_V = \frac{dU}{dT} = nR$$ //

Ⓐ: $\overline{E} = kT$, $U = 2493{,}11$ J, $C_V = 8{,}314$ J/K

Ⓑ: $\overline{E} = kT$, $U = 2493{,}11$ J, $C_V = 16{,}628$ J/K

Napomena: prosječna energija može se odrediti i pomoću particijske funkcije čestice:

$$\overline{E} = -\frac{\frac{\partial z}{\partial \beta}}{z}$$

$$z = \frac{2s+1}{h^f} \int e^{-\beta E} d\phi$$

$$\left.\begin{matrix} s = 0 \\ f = 1 \end{matrix}\right\} z = \frac{1}{h} \int_0^\infty e^{-\beta\left[\frac{p^2}{2m} + Ax^2\right]} dp\,dx$$

$$Z = \frac{1}{h}\left[\underbrace{\int_0^\infty e^{-\beta\frac{p^2}{2m}}dp}_{J_1}\underbrace{\int_0^\infty e^{-\beta A x^2}dx}_{J_2}\right]$$

$$J_1 = \int_0^\infty e^{-\left(\frac{\beta}{2m}\right)p^2}dp = I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\frac{\beta}{2m}}} = \frac{1}{2}\sqrt{\frac{2m\pi}{\beta}} \,//$$

$$J_2 = \int_0^\infty e^{-(\beta A)x^2}dx = I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\beta A}} \,//$$

$$Z = \frac{1}{h}\left(\frac{1}{2}\sqrt{\frac{2m\pi}{\beta}}\right)\left(\frac{1}{2}\sqrt{\frac{\pi}{\beta A}}\right)$$

$$Z = \frac{1}{h}\sqrt{\frac{\cancel{2}\, m\, \pi^2}{\underset{8}{\cancel{16}}\, \beta^2 A}}$$

$$\boxed{Z = \frac{1}{h}\sqrt{\frac{m}{8A}\cdot\frac{\pi}{\beta}}} \Big/ \frac{\partial}{\partial\beta}$$

$$\boxed{\frac{\partial Z}{\partial\beta} = \frac{\partial}{\partial\beta}\left[\frac{1}{h}\sqrt{\frac{m}{8A}}\,\pi\,\beta^{-1}\right] = -\frac{1}{h}\sqrt{\frac{m}{8A}}\,\frac{\pi}{\beta^2}}$$

$$\boxed{\bar{E} = -\frac{\frac{\partial Z}{\partial\beta}}{Z} = -\frac{-\frac{1}{h}\sqrt{\frac{m}{8A}}\,\frac{\pi}{\beta^2}}{\frac{1}{h}\sqrt{\frac{m}{8A}}\,\frac{\pi}{\beta}} = \frac{1}{\beta} = kT}$$

Ⓐ 3. Ⓑ 4.

Zadan: $g(E) = CVE^3, \quad E \geq 0$

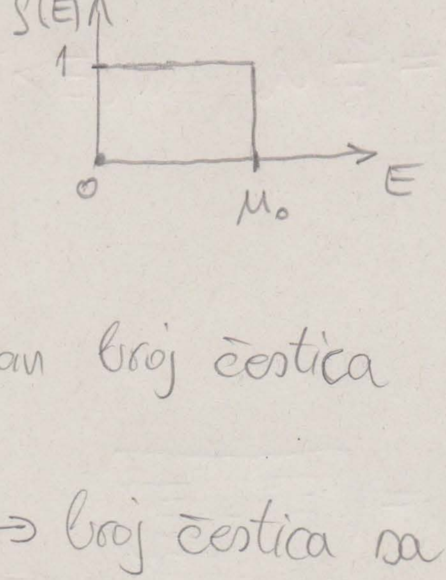
a) $T = 0K$

$w(E > \bar{E}) = ?$

$$w(E>\bar{E}) = \frac{\Delta N}{N} = \frac{N(E>\bar{E})}{N}$$

$$N = \int_0^\infty \underbrace{g(E)}_{\text{ZADANO}} \, \underbrace{\rho(E)} \, dE \rightarrow \text{za fermione na } T=0K:$$

$$\rho(E) = \begin{cases} 1, & E < \mu_0 \\ 0, & E > \mu_0 \end{cases}$$



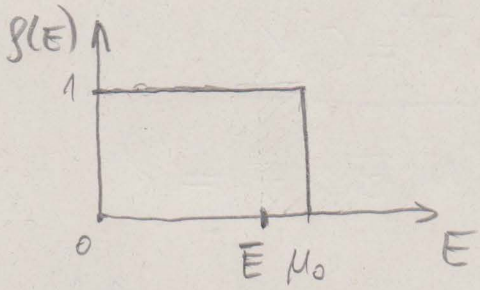
$N = \int_0^{\mu_0} CVE^3 dE \rightarrow$ ukupan broj čestica

$N(E>\bar{E}) = \int_{\bar{E}}^{\mu_0} CVE^3 dE \rightarrow$ broj čestica sa $E > \bar{E}$

$$w(E>\bar{E}) = \frac{\int_{\bar{E}}^{\mu_0} \cancel{CV} E^3 dE}{\int_0^{\mu_0} \cancel{CV} E^3 dE} = \frac{\frac{E^4}{\cancel{4}}\Big|_{\bar{E}}^{\mu_0}}{\frac{E^4}{\cancel{4}}\Big|_0^{\mu_0}} = \frac{\mu_0^4 - \bar{E}^4}{\mu_0^4}$$

$$\overline{E} = \frac{\int_0^\infty E\, g(E)\, \rho(E)\, dE}{\int_0^\infty g(E)\, \rho(E)\, dE} = \frac{\int_0^{\mu_0} E\, C V E^3\, dE}{\int_0^{\mu_0} C V E^3\, dE} = \frac{\int_0^{\mu_0} E^4\, dE}{\int_0^{\mu_0} E^3\, dE}$$

$$\overline{E} = \frac{\frac{E^5}{5}\Big|_0^{\mu_0}}{\frac{E^4}{4}\Big|_0^{\mu_0}} = \frac{4\mu_0^{\cancel{5}}}{5\cancel{\mu_0^4}} = \boxed{\frac{4}{5}\mu_0}$$



$$w(E>\overline{E}) = \frac{\mu_0^4 - \left(\frac{4}{5}\mu_0\right)^4}{\mu_0^4}$$

$$w(E>\overline{E}) = \frac{\cancel{\mu_0^4}\left(1-\left(\frac{4}{5}\right)^4\right)}{\cancel{\mu_0^4}} = 1-\left(\frac{4}{5}\right)^4 = \frac{369}{625}$$

$$\boxed{w(E>\overline{E}) = 0{,}5904}$$

Ⓐ, Ⓑ: $\overline{E} = \frac{4}{5}\mu_0$, $w(E>\overline{E}) = 0{,}5904$

b) $\Delta E = ?$

$\frac{\Delta E}{\overline{E}} = ?$

$$\Delta E = \sigma(E) = \sqrt{\overline{E^2} - \overline{E}^2}$$

$$(\overline{E})^2 = \left(\frac{4}{5}\mu_0\right)^2 = \frac{16}{25}\mu_0^2$$

$$\overline{E^2} = \frac{\int_0^\infty E^2 g(E)\, \rho(E)\, dE}{\int_0^\infty g(E)\, \rho(E)\, dE} = \frac{\int_0^{\mu_0} E^2 C V E^3\, dE}{\int_0^{\mu_0} C V E^3\, dE} = \frac{\int_0^{\mu_0} E^5\, dE}{\int_0^{\mu_0} E^3\, dE}$$

$$\overline{E^2} = \frac{\frac{E^6}{6}\Big|_0^{\mu_0}}{\frac{E^4}{4}\Big|_0^{\mu_0}} = \frac{4\mu_0^{\cancel{6}}}{6\cancel{\mu_0^4}} = \frac{4}{6}\mu_0^2$$

$$\Delta E = \sqrt{\frac{4}{6}\mu_0^2 - \frac{16}{25}\mu_0^2} = \sqrt{\frac{100-96}{150}}\,\mu_0 = \frac{2}{\sqrt{150}}\mu_0 = \frac{2}{5\sqrt{6}}\mu_0 \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$\boxed{\Delta E = \frac{\sqrt{6}}{15}\mu_0}$$

$$\boxed{\frac{\Delta E}{E} = \frac{\frac{\sqrt{6}}{\overset{3}{\cancel{15}}}\cancel{\mu_0}}{\frac{4}{\cancel{5}}\cancel{\mu_0}} = \frac{\sqrt{6}}{12}}$$

Ⓐ, Ⓑ: $\Delta E = \frac{\sqrt{6}}{15}\mu_0$, $\frac{\Delta E}{E} = \frac{\sqrt{6}}{12}$

Ⓐ 4. Ⓑ 1.

A:
$W(v_m < v_x < \bar{v}) = ?$

B:
$w(v_m < v_x < v_{sk}) = ?$

Razlika A i B grupe je bila samo u gornjoj granici

$$v_m = \sqrt{\frac{2kT}{m_0}}$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m_0}}$$

$$dw_x = f(v_x^2)\,dv_x \,/ \int$$

$$w_x = \int f(v_x^2)\,dv_x$$

$$f(v_x^2) = \frac{e^{-\frac{m_0 v_x^2}{2kT}}}{\underbrace{\int_0^\infty e^{-\frac{m_0 v_x^2}{2kT}}\,dv_x}}$$

$$J = \int_0^\infty e^{-\left(\frac{m_0}{2kT}\right) v_x^2}\,dv_x = I_0$$

$$J = \frac{1}{2}\sqrt{\frac{\pi}{a}} = \frac{1}{2}\sqrt{\frac{\pi}{\frac{m_0}{2kT}}}$$

$$J = \frac{1}{2}\sqrt{\frac{2\pi kT}{m_0}}$$

$$\boxed{f(v_x^2) = 2\sqrt{\frac{m_0}{2\pi kT}}\, e^{-\frac{m_0 v_x^2}{2kT}}}$$

$$w_x = \int_{v_m}^{\bar{v}} 2\sqrt{\frac{m_0}{2\pi kT}}\, e^{-\frac{m_0 v_x^2}{2kT}}\,dv_x$$

$$w_x = 2\sqrt{\frac{m_0}{2\pi kT}} \int_{v_m}^{\bar{v}} e^{-\frac{m_0 v_x^2}{2kT}}\, dv_x$$

$\Longrightarrow$ ovaj integral rješavamo supstitucijom:

$$t^2 = \frac{m_0}{2kT} v_x^2 / \sqrt{\ } \longrightarrow v_x^2 = \frac{2kT}{m_0} t^2$$

$$t = \sqrt{\frac{m_0}{2kT}}\, v_x \longrightarrow v_x = \sqrt{\frac{2kT}{m_0}}\, t / d$$

$$\boxed{dv_x = \sqrt{\frac{2kT}{m_0}}\, dt}$$

PROMJENA GRANICA:

$$t_1 = \sqrt{\frac{m_0}{2kT}}\, v_m = \sqrt{\frac{\cancel{m_0}}{\cancel{2kT}}} \cdot \sqrt{\frac{\cancel{2kT}}{\cancel{m_0}}} = 1$$

$$t_2 = \sqrt{\frac{m_0}{2kT}}\, \bar{v} = \sqrt{\frac{\cancel{m_0}}{\cancel{2kT}}} \cdot \sqrt{\frac{8\cancel{kT}}{\pi \cancel{m_0}}} = \sqrt{\frac{4}{\pi}} = 1,128$$

$$w_x = 2\sqrt{\frac{m_0}{2\pi kT}} \int_{t_1}^{t_2} e^{-t^2} \sqrt{\frac{2kT}{m_0}}\, dt$$

$$w_x = 2\sqrt{\frac{\cancel{m_0}}{2\pi \cancel{kT}} \cdot \frac{2\cancel{kT}}{\cancel{m_0}}} \int_{t_1}^{t_2} e^{-t^2}\, dt$$

$$w_x = \frac{2}{\sqrt{\pi}} \int_{t_1}^{t_2} e^{-t^2}\, dt$$

$$w_x = \frac{2}{\sqrt{\pi}} \int_{0}^{t_2} e^{-t^2}\, dt - \frac{2}{\sqrt{\pi}} \int_{0}^{t_1} e^{-t^2}\, dt$$

$$w_x = \underbrace{\frac{2}{\sqrt{\pi}}\int_0^{t_2} e^{-t^2}dt}_{erf(t_2)} - \underbrace{\frac{2}{\sqrt{\pi}}\int_0^{t_1} e^{-t^2}dt}_{erf(t_1)}$$

$$w_x = erf(1{,}128) - erf(1)$$

$$\boxed{w_x = 0{,}046634}$$

*U B grupi je gornja granica v_{sk} a ne $\bar{v}$, pa se ona supstituira pri računanju nove gornje granice:

$$v_{sk} = \sqrt{\frac{3kT}{m_0}}$$

$$t_2 = \sqrt{\frac{m_0}{2kT}}\cdot v_{sk} = \sqrt{\frac{\cancel{m_0}}{2\cancel{kT}}}\cdot\sqrt{\frac{3\cancel{kT}}{\cancel{m_0}}} = \sqrt{\frac{3}{2}} = 1{,}225$$

⋮

$$w_x = erf(1{,}225) - erf(1)$$

$$\boxed{w_x = 0{,}07409}$$

Ⓐ: $w(v_m < v_x < \bar{v}) = 0{,}04663$

Ⓑ: $w(v_m < v_x < v_{sk}) = 0{,}07409$